

Grammar and Automata Correspondence

Grammar

Automaton

Regular Grammar

Finite-State Automaton

Context-Free Grammar

Push-Down Automaton

Context-Sensitive
Grammar

Turing Machine

CFG Versus PDA

- CFGs and PDAs are of equivalent power
- Grammar Implementation Mechanism:
 - Translate CFG to PDA, then use PDA to parse input string
 - Foundation for bottom-up parser generators

Pushdown Automata

- Consists of
 - Pushdown stack (can have terminals and nonterminals)
 - Finite state automaton control
- Can do one of three actions (based on state and input):
 - Shift:
 - Shift current input symbol from input onto stack
 - Reduce:
 - If symbols on top of stack match right hand side of some grammar production $NT \rightarrow \beta$
 - Pop symbols (β) off of the stack
 - Push left hand side nonterminal (NT) onto stack
 - Accept the input string

Shift-Reduce Parser Example

Stack

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow (Expr)$

$Expr \rightarrow - Expr$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$

Input String

num	*	(num	+	num)
-----	---	---	-----	---	-----	---

Shift-Reduce Parser Example

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow (Expr)$

$Expr \rightarrow - Expr$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$

num	*	(num	+	num)
-----	---	---	-----	---	-----	---

Shift-Reduce Parser Example

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow (Expr)$

$Expr \rightarrow - Expr$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$

SHIFT

num	*	(num	+	num)
-----	---	---	-----	---	-----	---

Shift-Reduce Parser Example

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow (Expr)$

$Expr \rightarrow - Expr$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$

num

SHIFT

*	(num	+	num)
---	---	-----	---	-----	---

Shift-Reduce Parser Example

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow (Expr)$

$Expr \rightarrow - Expr$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$

num

REDUCE

*	(num	+	num)
---	---	-----	---	-----	---

Shift-Reduce Parser Example

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow (Expr)$

$Expr \rightarrow - Expr$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$

Expr

REDUCE

num

*	(num	+	num)
---	---	-----	---	-----	---

Shift-Reduce Parser Example

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow (Expr)$

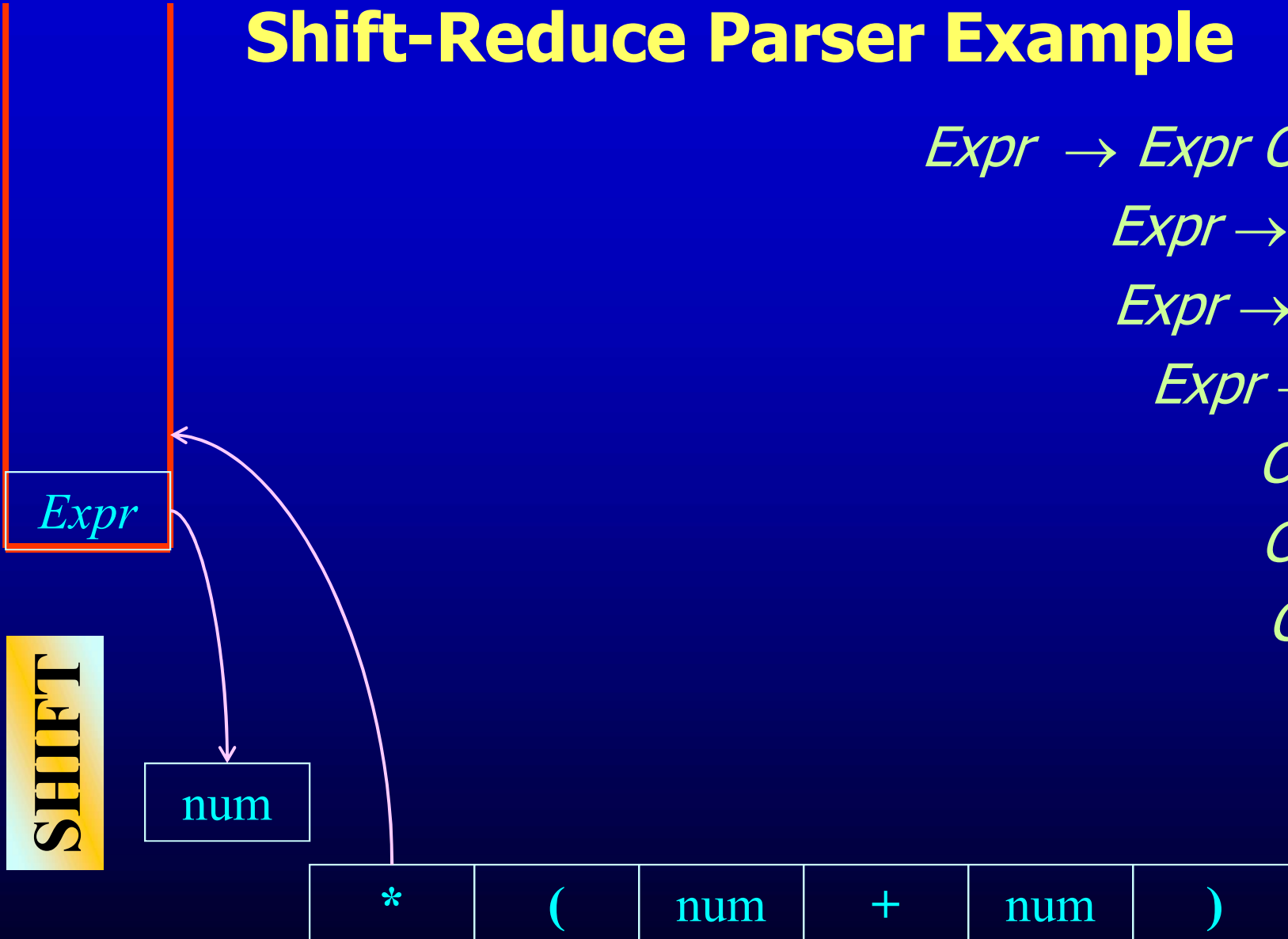
$Expr \rightarrow - Expr$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$



Shift-Reduce Parser Example

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow (Expr)$

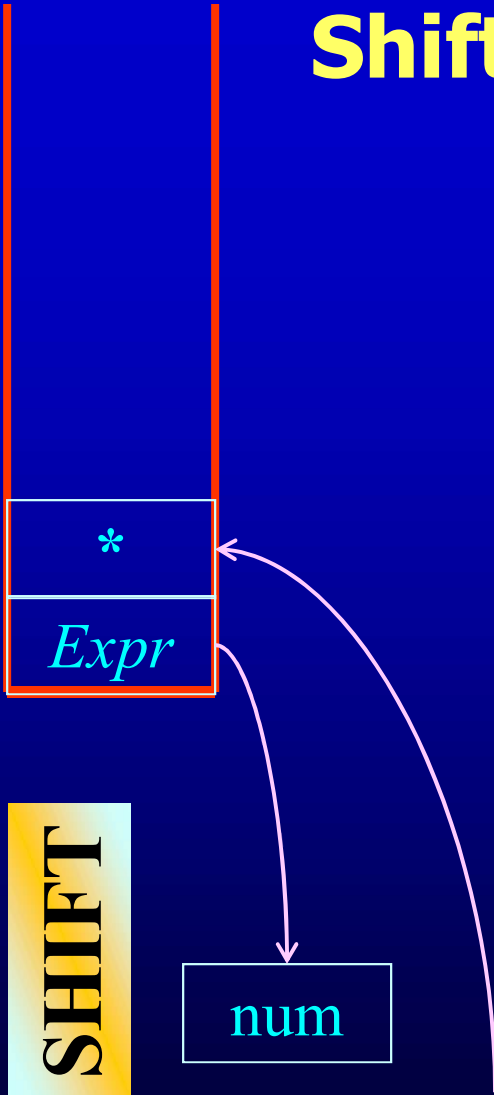
$Expr \rightarrow - Expr$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$



(num	+	num)
---	-----	---	-----	---

Shift-Reduce Parser Example

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow (Expr)$

$Expr \rightarrow - Expr$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$

Op
Expr

REDUCE

num *

(num + num)

Shift-Reduce Parser Example

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow (Expr)$

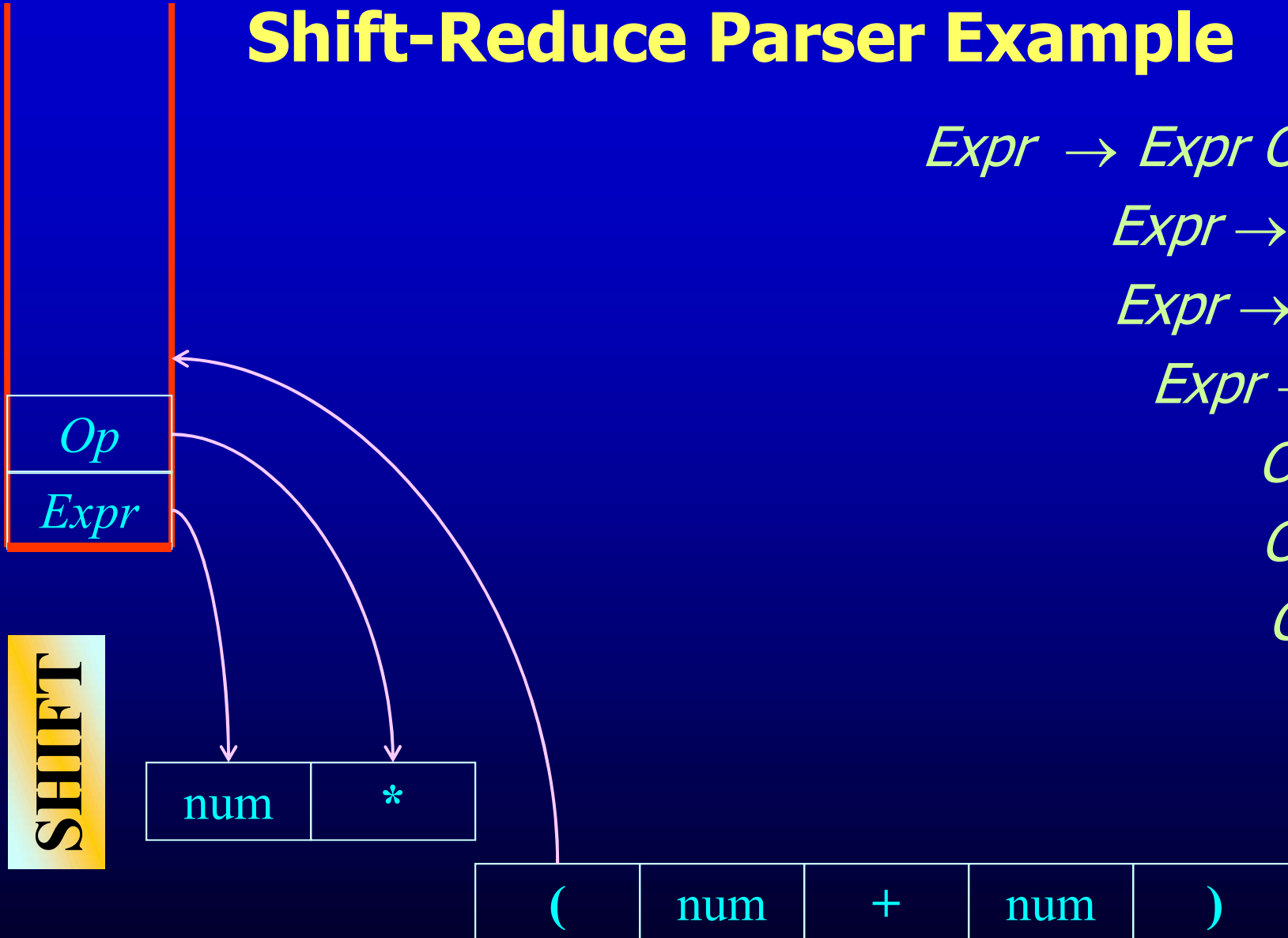
$Expr \rightarrow - Expr$

$Expr \rightarrow num$

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$Op \rightarrow -$

$Op \rightarrow *$



Shift-Reduce Parser Example

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow (Expr)$

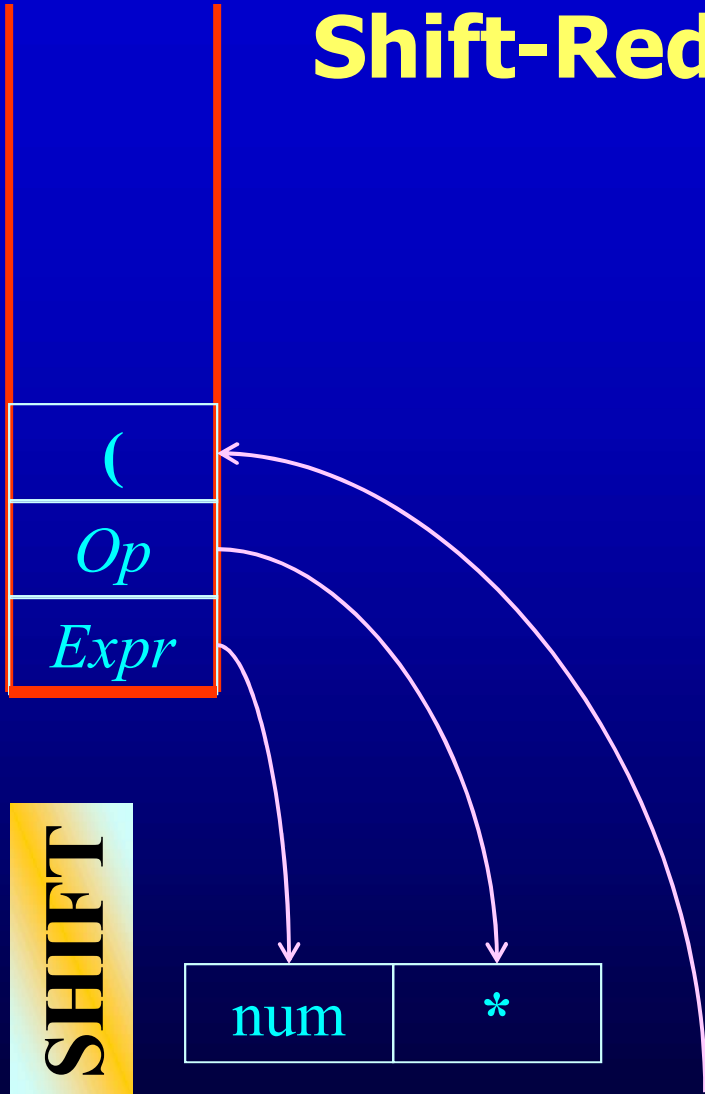
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Shift-Reduce Parser Example

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow (Expr)$

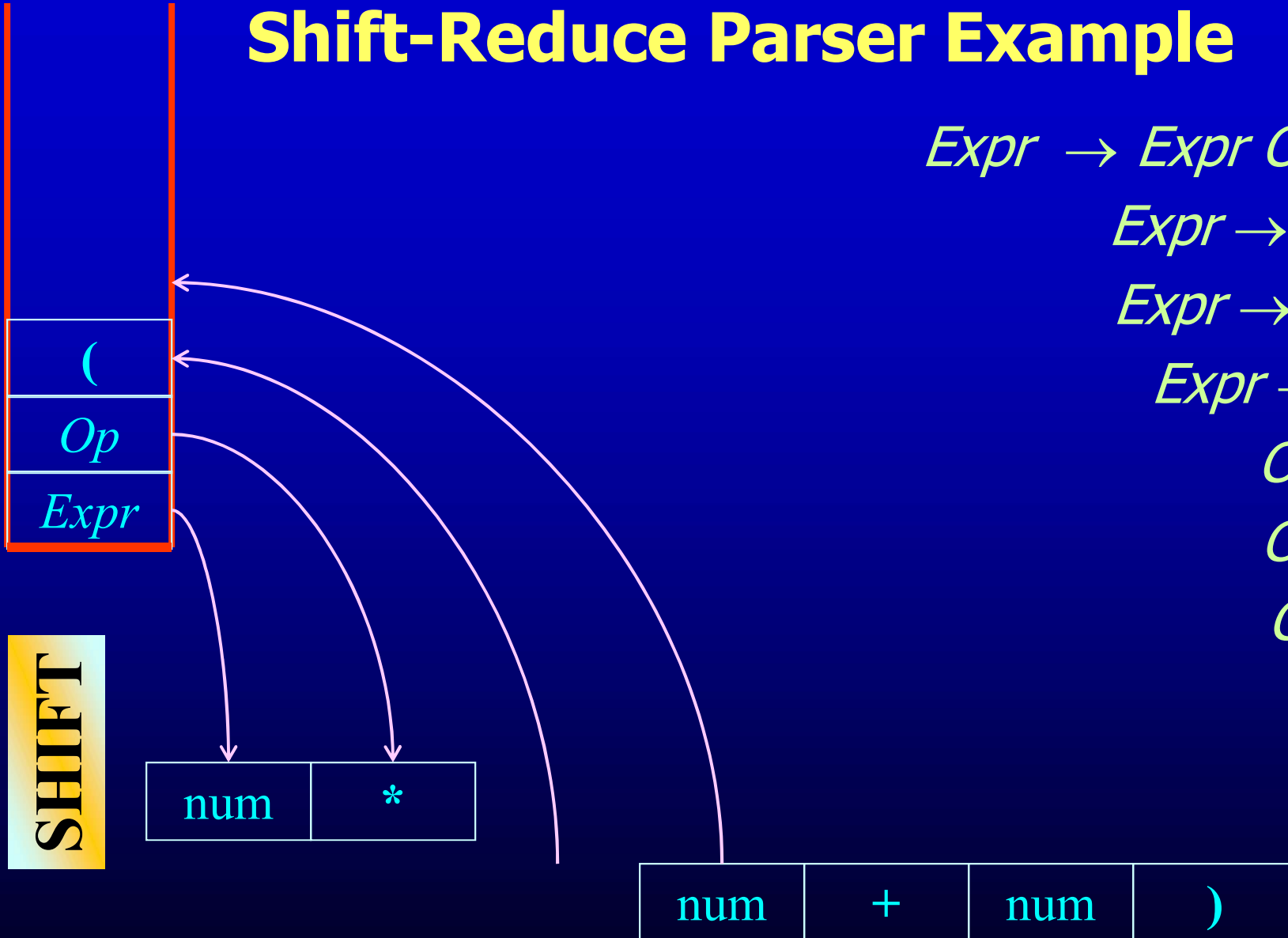
$Expr \rightarrow - Expr$

$Expr \rightarrow num$

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$Op \rightarrow -$

$Op \rightarrow *$



Shift-Reduce Parser Example

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow (Expr)$

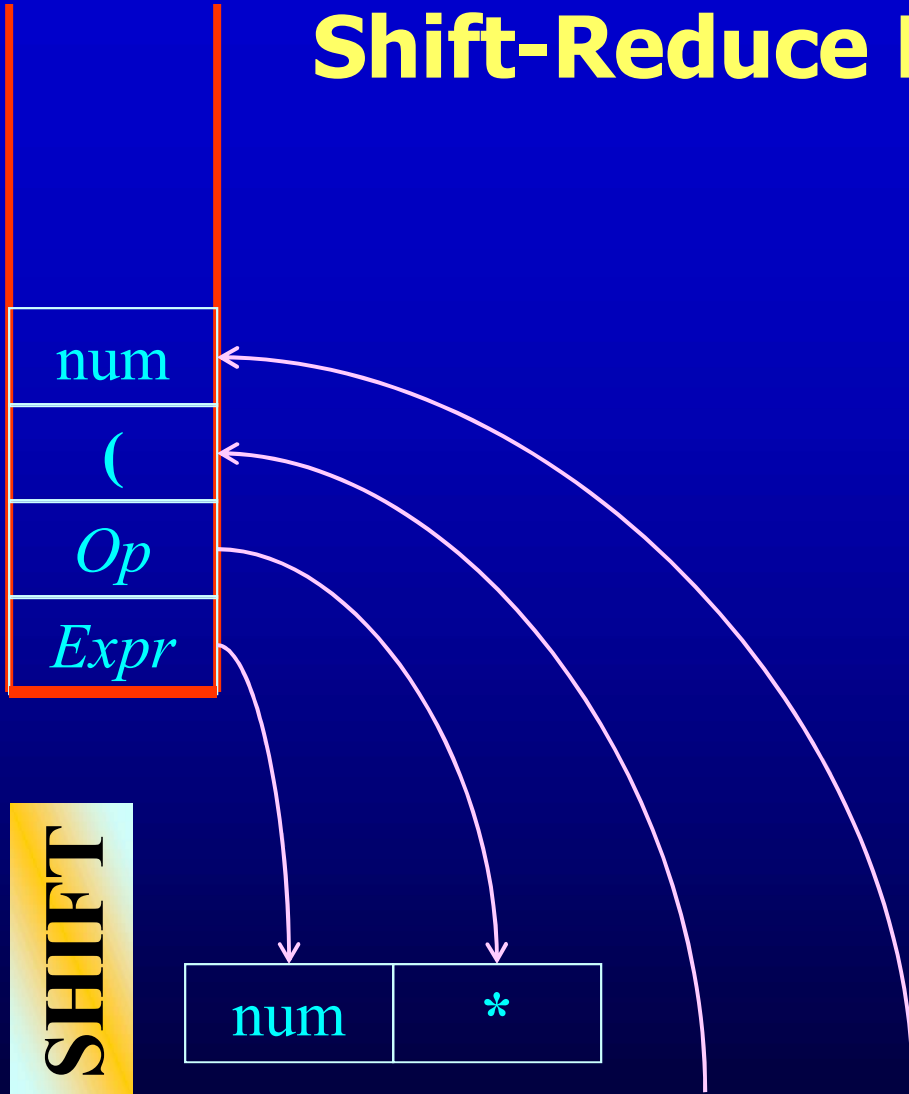
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$Expr \rightarrow num$

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$Op \rightarrow -$

$Op \rightarrow *$



Shift-Reduce Parser Example

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow (Expr)$

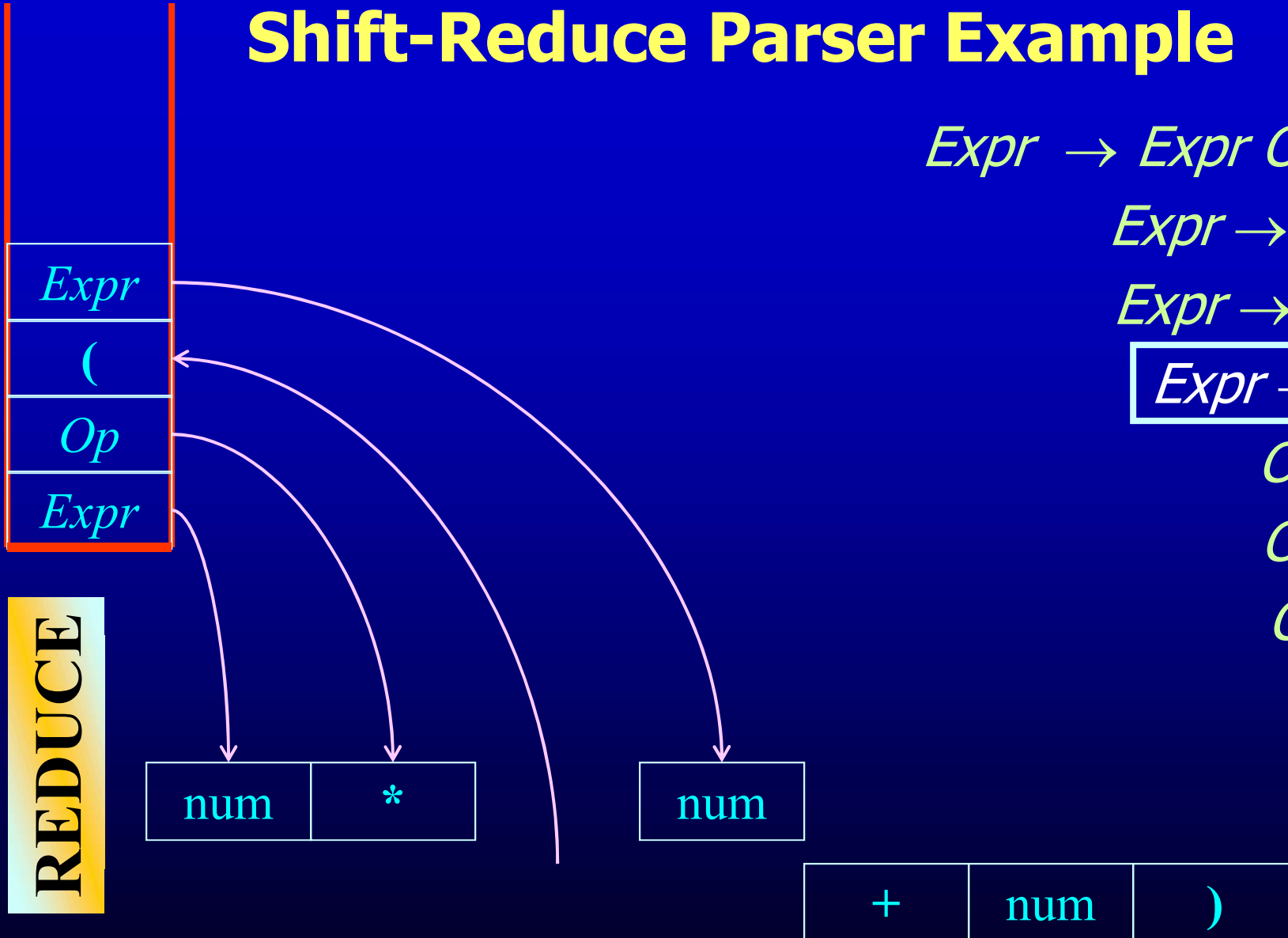
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$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$



Shift-Reduce Parser Example

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow (Expr)$

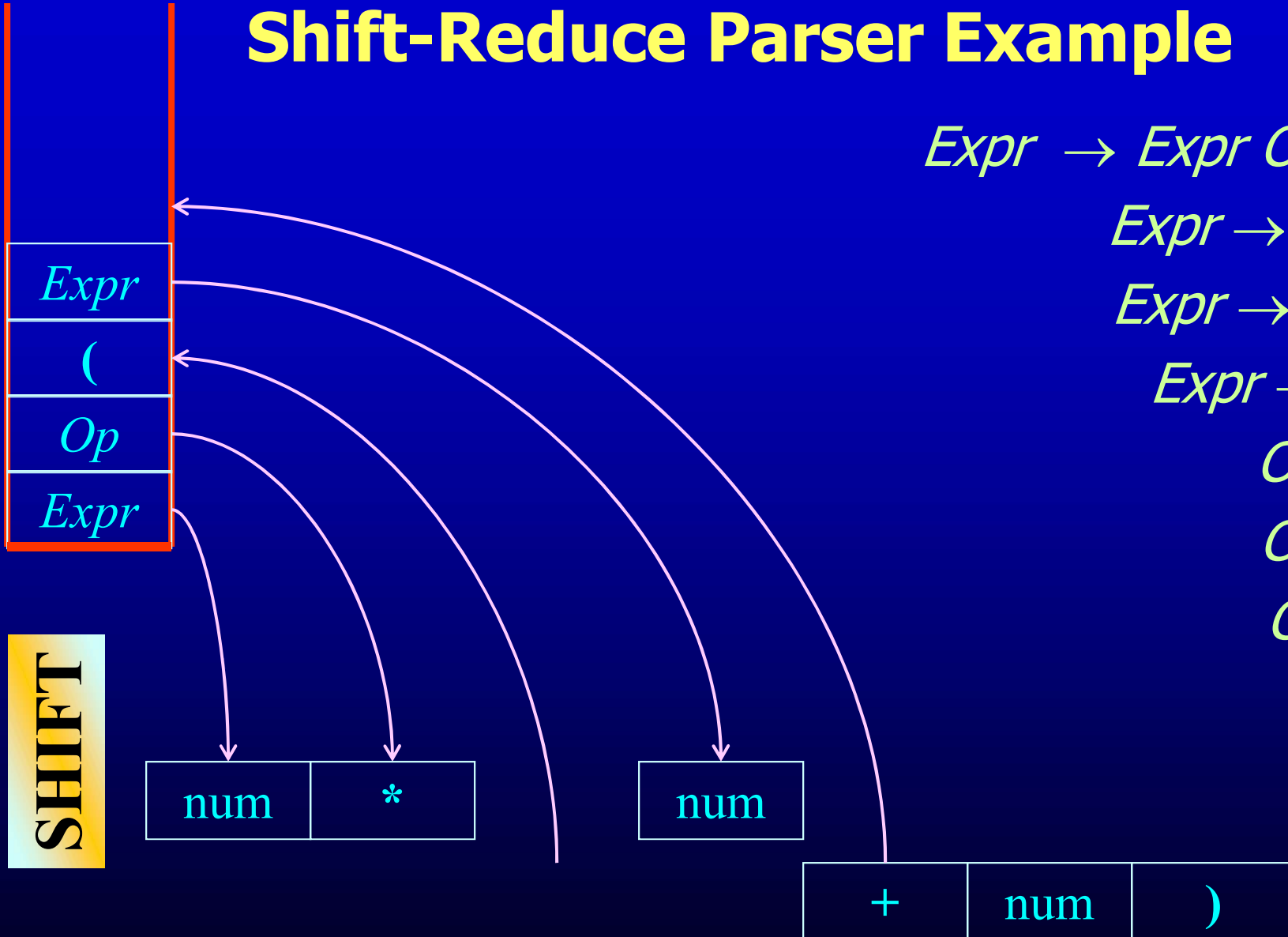
$Expr \rightarrow - Expr$

$Expr \rightarrow num$

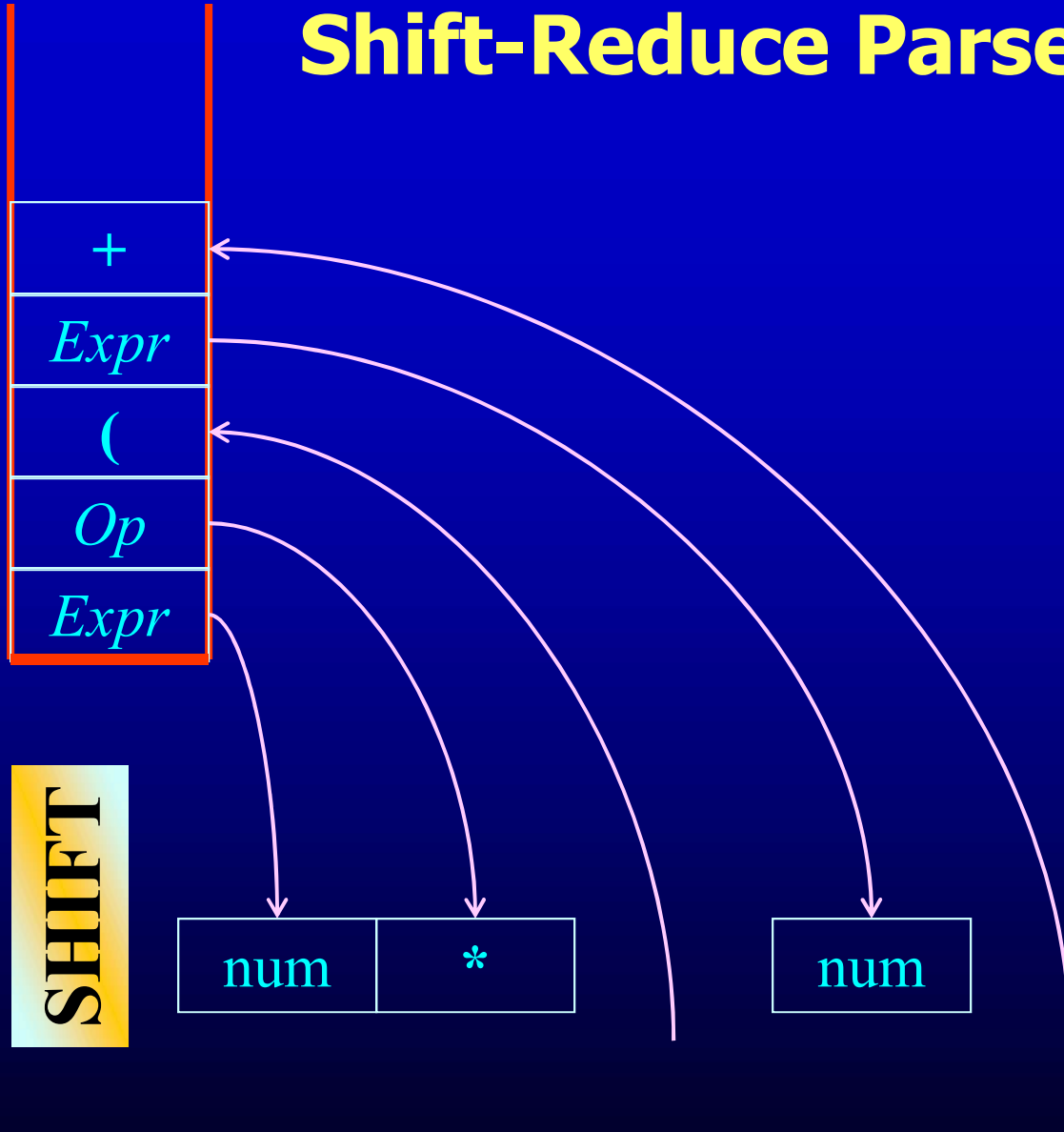
$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$



Shift-Reduce Parser Example



$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow (Expr)$

$Expr \rightarrow - Expr$

$Expr \rightarrow num$

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Shift-Reduce Parser Example

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow (Expr)$

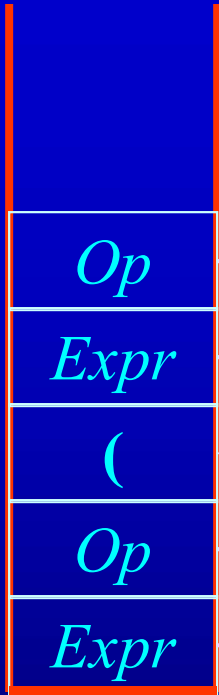
$Expr \rightarrow - Expr$

$Expr \rightarrow num$

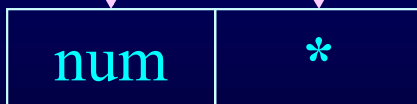
$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$



REDUCE



Shift-Reduce Parser Example

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow (Expr)$

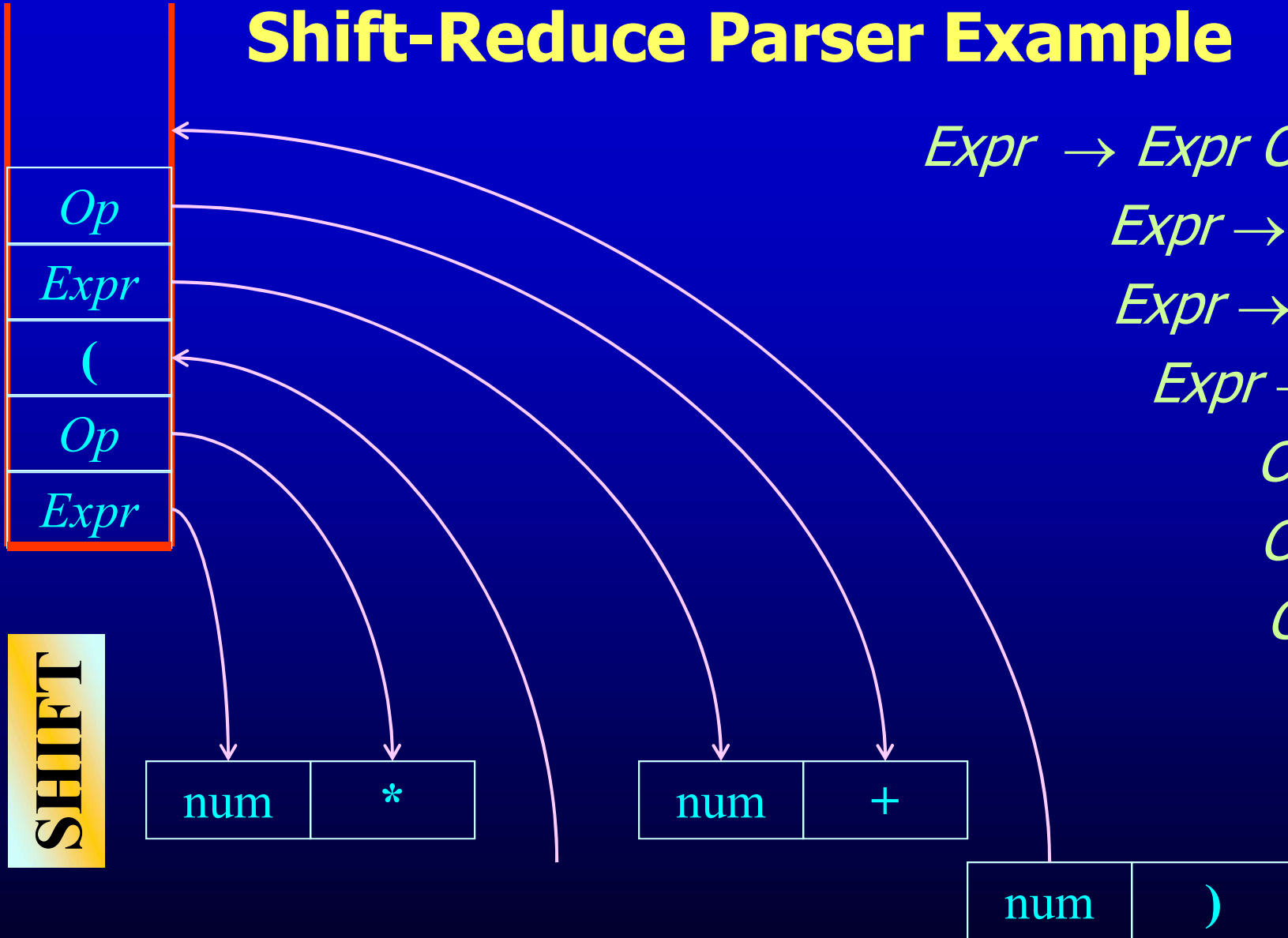
$Expr \rightarrow - Expr$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$



Shift-Reduce Parser Example



$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow (Expr)$

$Expr \rightarrow - Expr$

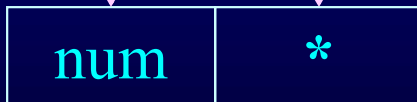
$Expr \rightarrow num$

$Op \rightarrow +$

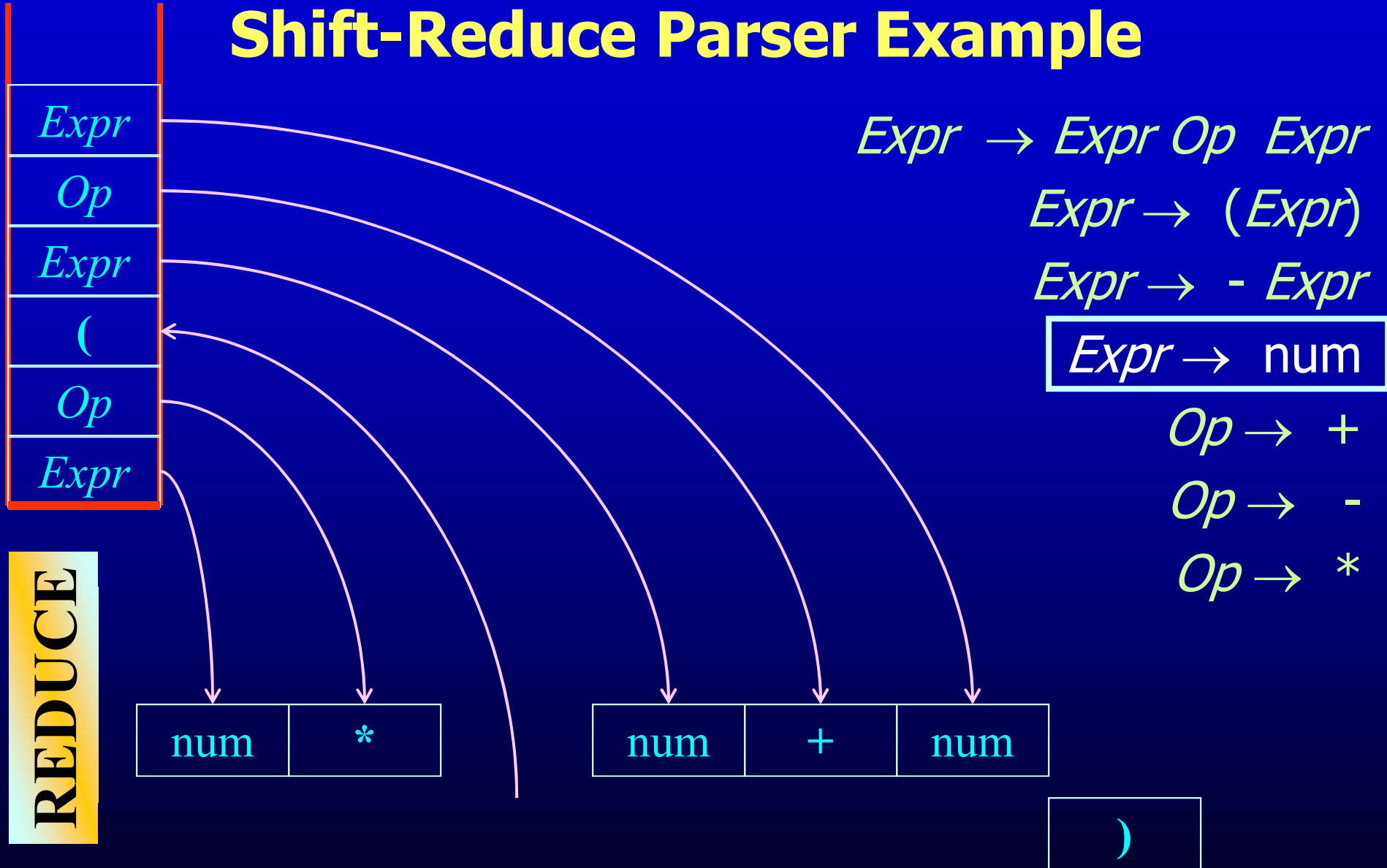
$Op \rightarrow -$

$Op \rightarrow *$

SHIFT



Shift-Reduce Parser Example



Shift-Reduce Parser Example

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow (Expr)$

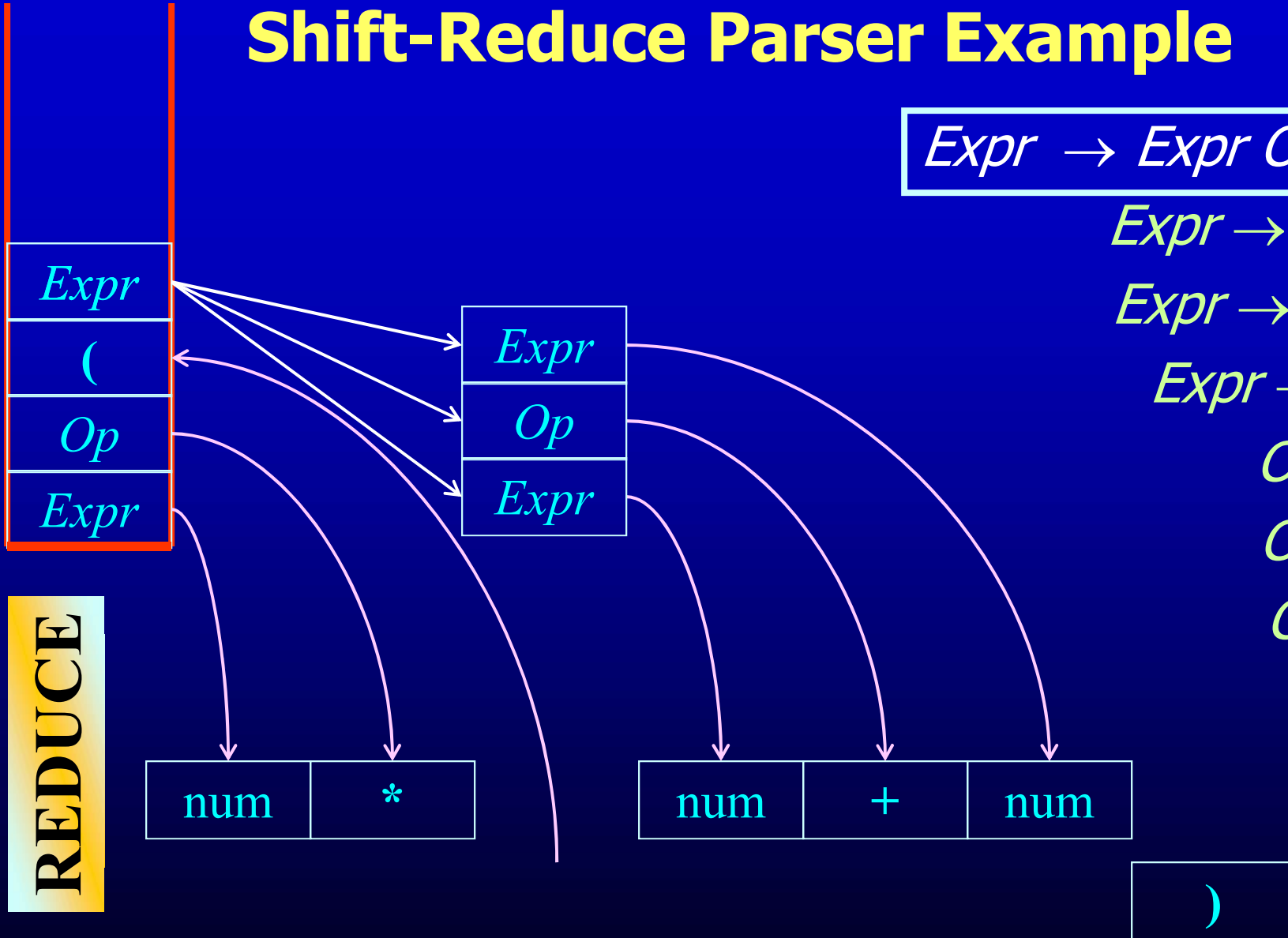
$Expr \rightarrow - Expr$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$



Shift-Reduce Parser Example

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow (Expr)$

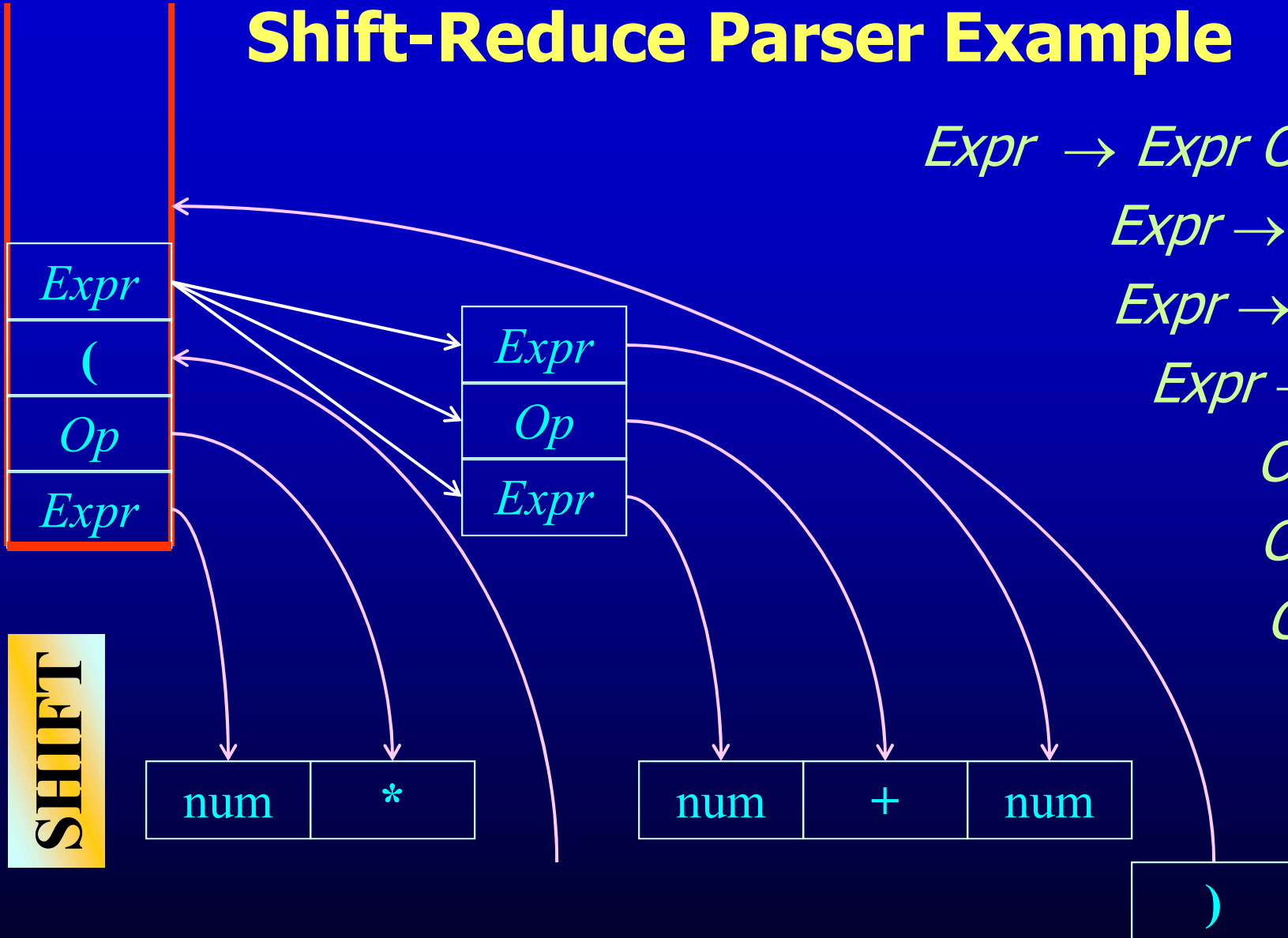
$Expr \rightarrow - Expr$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$



Shift-Reduce Parser Example

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow (Expr)$

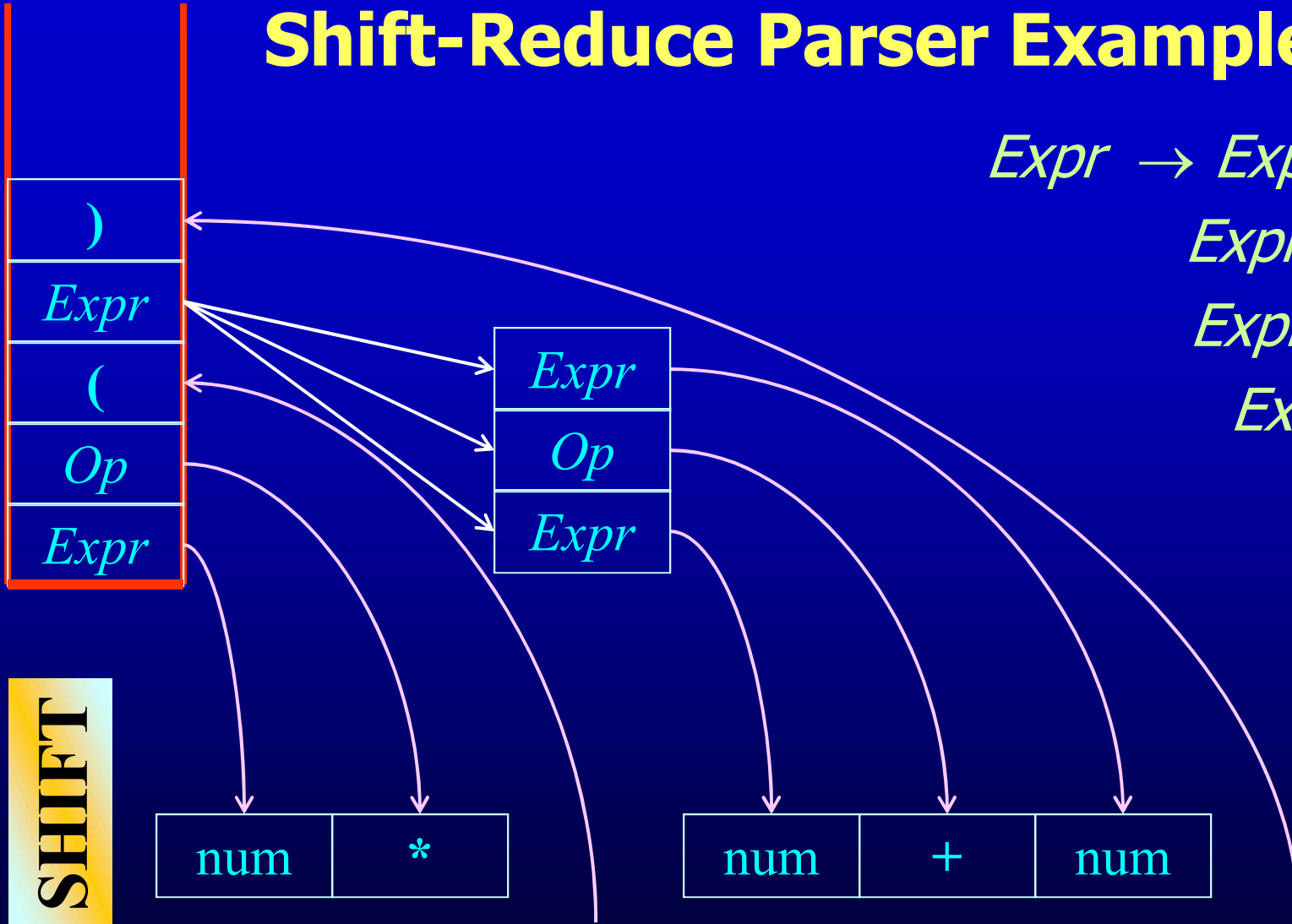
$Expr \rightarrow - Expr$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$



Shift-Reduce Parser Example

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow (Expr)$

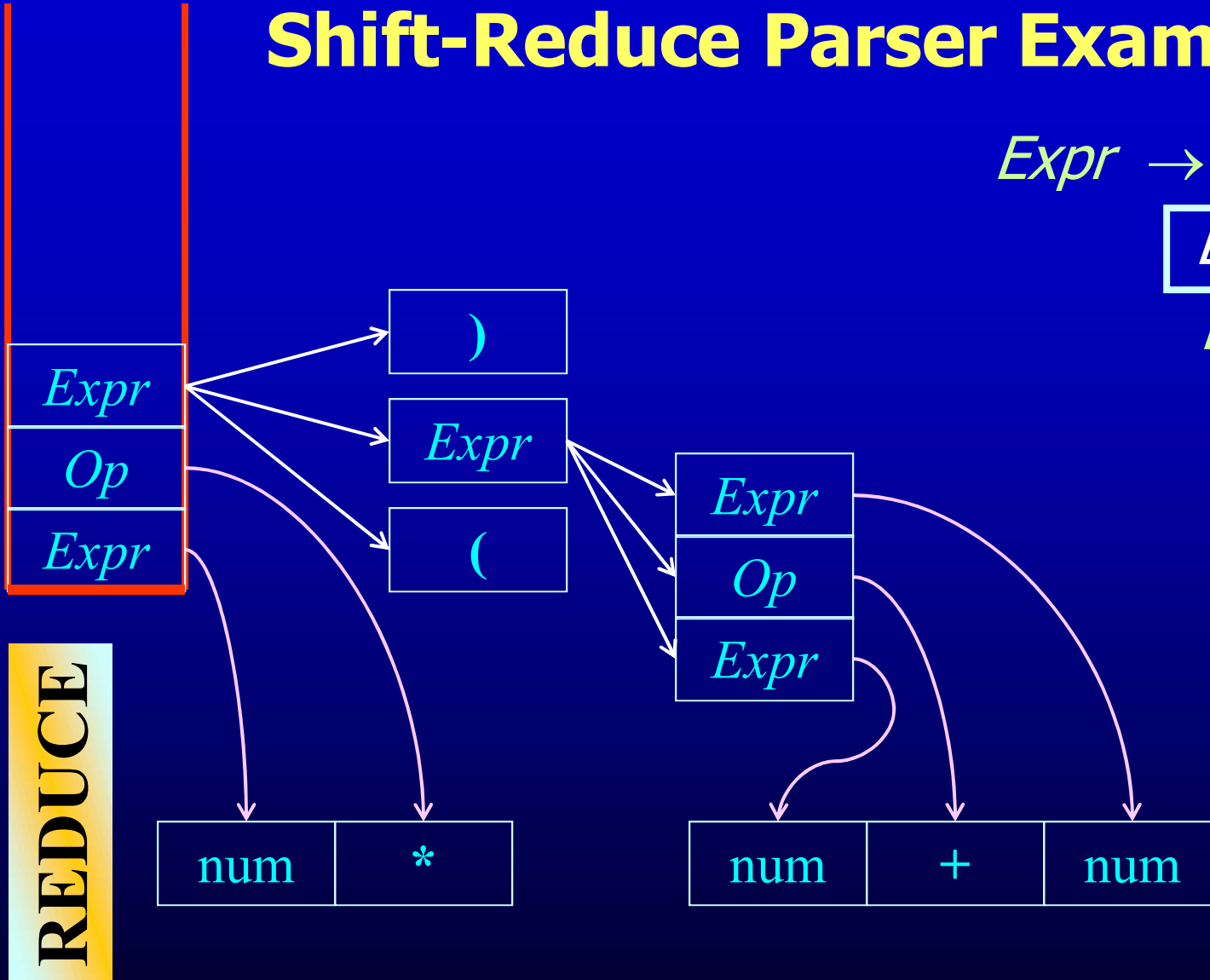
$Expr \rightarrow - Expr$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$



Shift-Reduce Parser Example

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow (Expr)$

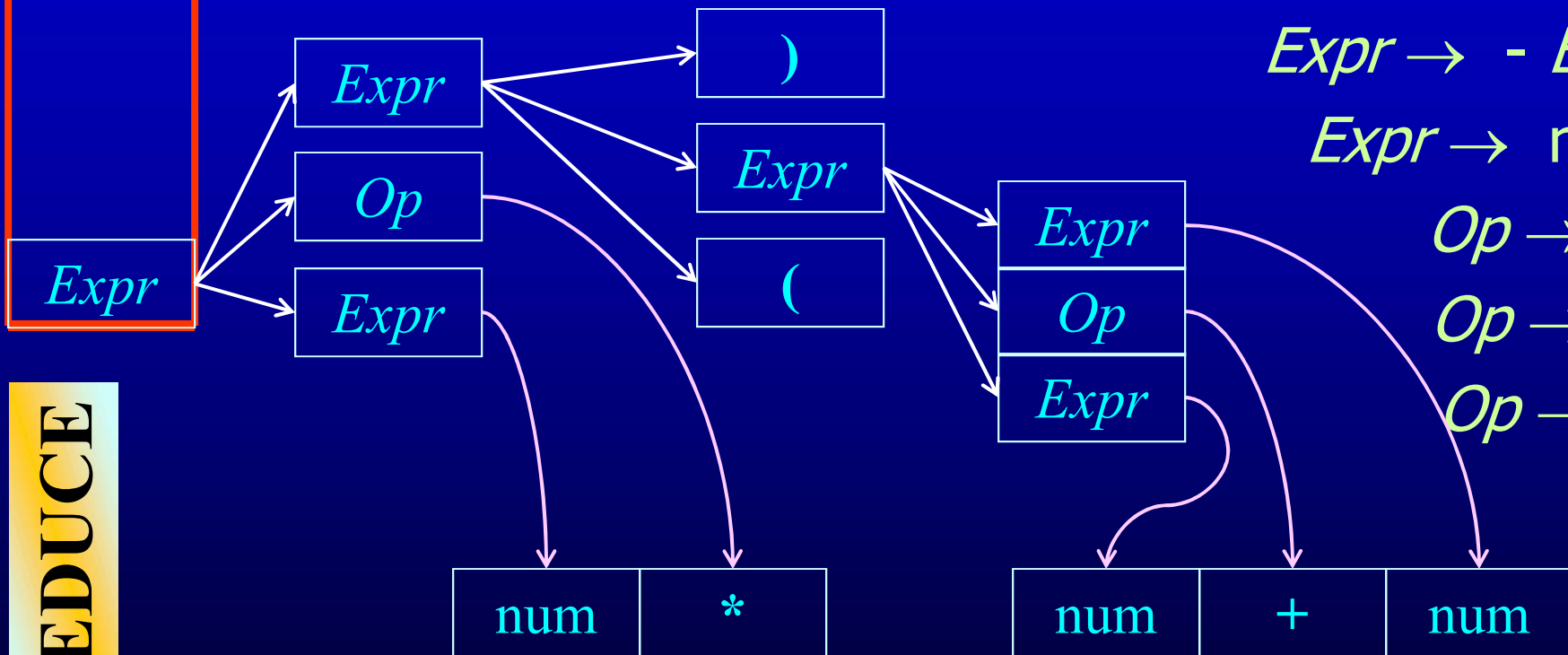
$Expr \rightarrow - Expr$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$



REDUCE

Shift-Reduce Parser Example

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow (Expr)$

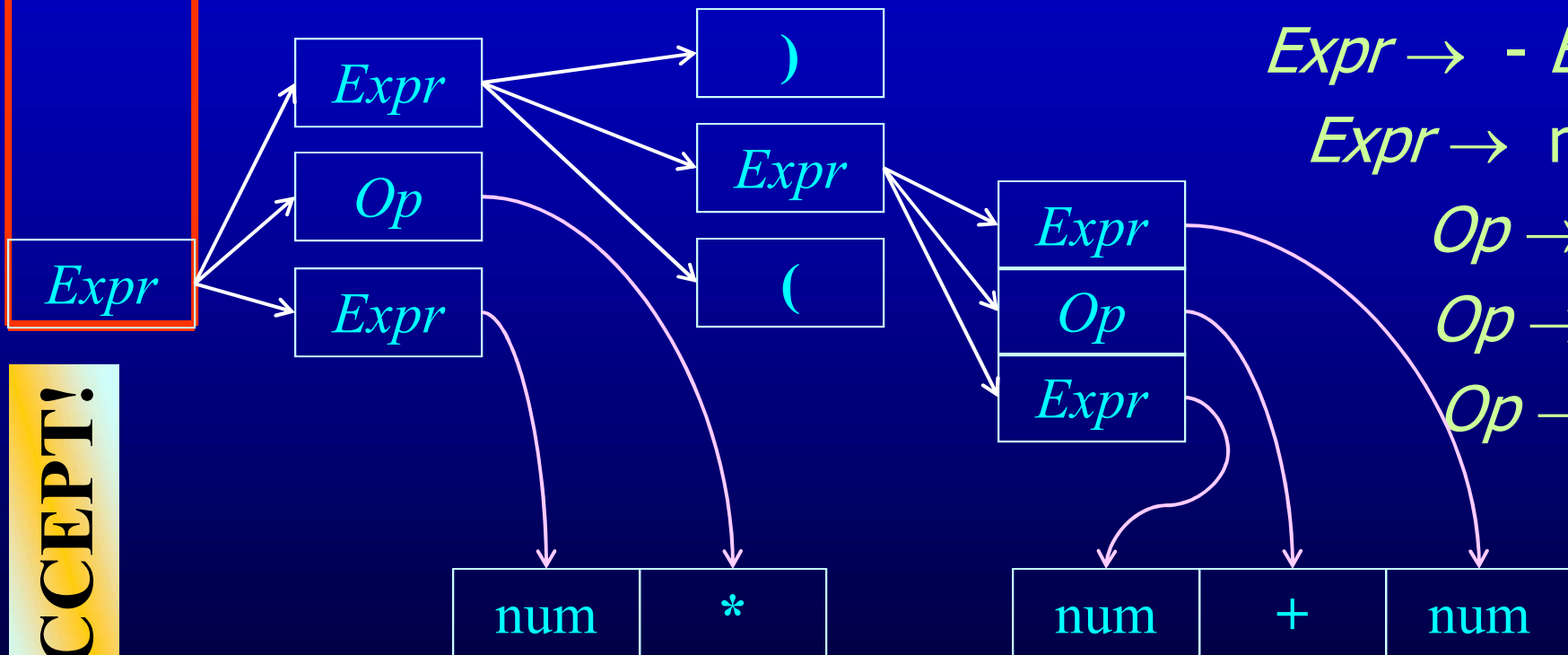
$Expr \rightarrow - Expr$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$



Basic Idea

- Goal: reconstruct parse tree for input string
- Read input from left to right
- Build tree in a bottom-up fashion
 - Apply production rules backwards (RHS \rightarrow LHS)
- Build tree from right to left (LR parser)
- Use stack to hold pending sequences of terminals and nonterminals

Outline

- Bottom-Up parsing
- **Conflicts**
- Constructing a Shift-Reduce Parser
- Parser and syntax trees

Potential Conflicts

- Reduce/Reduce Conflict
 - Top of the stack may match RHS of multiple productions
 - Which production to use in the reduction?
- Shift/Reduce Conflict
 - Stack may match RHS of production
 - But that may not be the right match
 - May need to shift an input and later find a different reduction

Conflicts

- Original Grammar

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow (Expr)$

$Expr \rightarrow - Expr$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$

- New Grammar

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow Expr - Expr$

$Expr \rightarrow (Expr)$

$Expr \rightarrow Expr -$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$

Conflicts

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow Expr - Expr$

$Expr \rightarrow (Expr)$

$Expr \rightarrow Expr -$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$

num	-	num
-----	---	-----

Conflicts

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow Expr - Expr$

$Expr \rightarrow (Expr)$

$Expr \rightarrow Expr -$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$

SHIFT

num	-	num
-----	---	-----



Conflicts

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow Expr - Expr$

$Expr \rightarrow (Expr)$

$Expr \rightarrow Expr -$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$

num

SHIFT

-

num

Conflicts

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow Expr - Expr$

$Expr \rightarrow (Expr)$

$Expr \rightarrow Expr -$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$

Expr

REDUCE

num

-

num

Conflicts

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow Expr - Expr$

$Expr \rightarrow (Expr)$

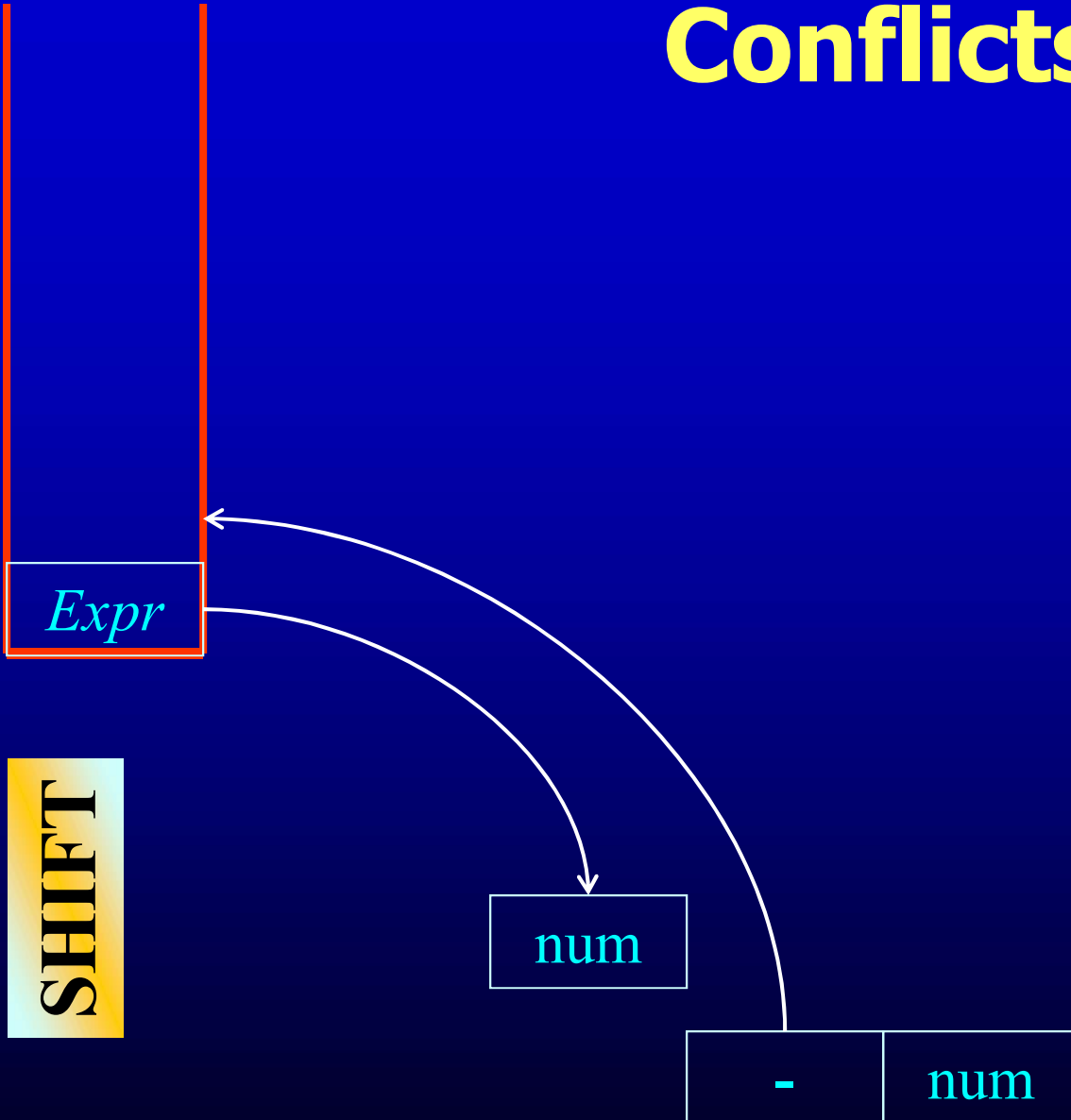
$Expr \rightarrow Expr -$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$



Conflicts

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow Expr - Expr$

$Expr \rightarrow (Expr)$

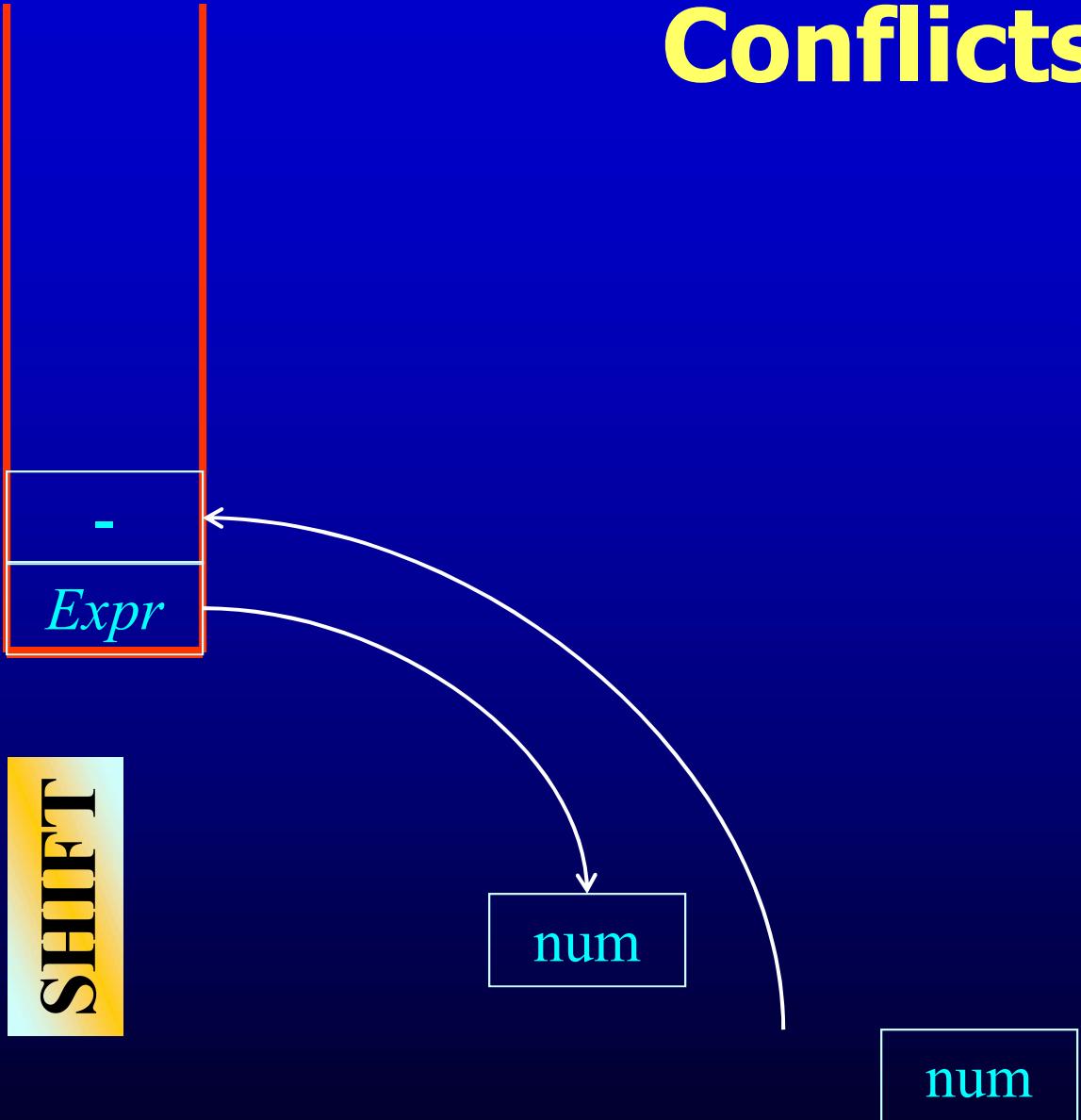
$Expr \rightarrow Expr -$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$



Shift/Reduce/Reduce Conflict

Options:

Reduce

Reduce

Shift

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow Expr - Expr$

$Expr \rightarrow (Expr)$

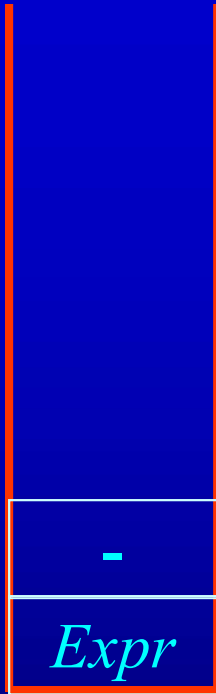
$Expr \rightarrow Expr -$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$



Shift/Reduce/Reduce Conflict

What Happens
if Choose
Reduce

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow Expr - Expr$

$Expr \rightarrow (Expr)$

$Expr \rightarrow Expr -$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$

-
Expr

REDUCE

num

num

Shift/Reduce/Reduce Conflict

What Happens
if Choose
Reduce

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow Expr - Expr$

$Expr \rightarrow (Expr)$

$Expr \rightarrow Expr -$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$

Expr

Expr

num -

num

SHIFT

Shift/Reduce/Reduce Conflict

What Happens
if Choose
Reduce

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow Expr - Expr$

$Expr \rightarrow (Expr)$

$Expr \rightarrow Expr -$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$

num
Expr

Expr

num -

SHIFT

Shift/Reduce/Reduce Conflict

What Happens
if Choose
Reduce

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow Expr - Expr$

$Expr \rightarrow (Expr)$

$Expr \rightarrow Expr -$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$

$Expr$
 $Expr$

$Expr$

num	-	num
-----	---	-----

REDUCE

Shift/Reduce/Reduce Conflict

What Happens
if Choose
Reduce

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow Expr - Expr$

$Expr \rightarrow (Expr)$

$Expr \rightarrow Expr -$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$

$Expr$
 $Expr$

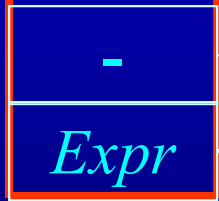
$Expr$

num	-	num
-----	---	-----

FAILS!

Shift/Reduce/Reduce Conflict

Both of These
Actions Work



Reduce
Shift

num

num

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow Expr - Expr$

$Expr \rightarrow (Expr)$

$Expr \rightarrow Expr -$

$Expr \rightarrow num$

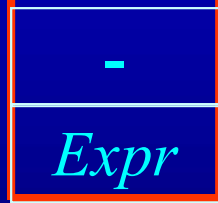
$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$

Shift/Reduce/Reduce Conflict

What Happens
if Choose



Reduce

num

num

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow Expr - Expr$

$Expr \rightarrow (Expr)$

$Expr \rightarrow Expr -$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$

Shift/Reduce/Reduce Conflict

What Happens
if Choose

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow Expr - Expr$

$Expr \rightarrow (Expr)$

$Expr \rightarrow Expr -$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$

Reduce

Op
Expr

REDUCE

num -

num

Shift/Reduce/Reduce Conflict

What Happens
if Choose



Reduce



$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow Expr - Expr$

$Expr \rightarrow (Expr)$

$Expr \rightarrow Expr -$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$



Shift/Reduce/Reduce Conflict

What Happens
if Choose

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow Expr - Expr$

$Expr \rightarrow (Expr)$

$Expr \rightarrow Expr -$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$

Expr
Op
Expr

Reduce

REDUCE

num - num

Shift/Reduce/Reduce Conflict

What Happens
if Choose

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow Expr - Expr$

$Expr \rightarrow (Expr)$

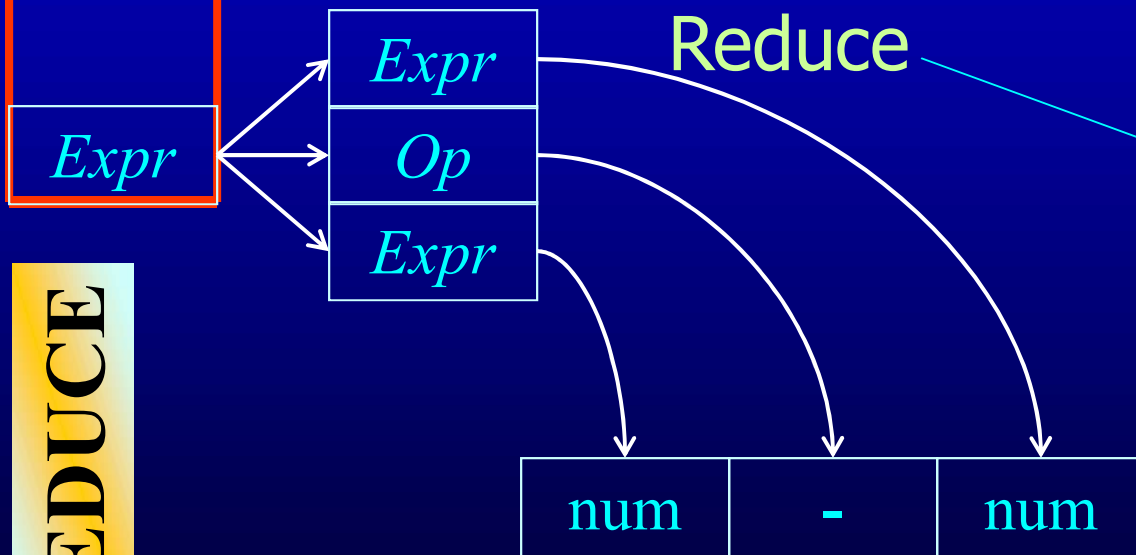
$Expr \rightarrow Expr -$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$



Shift/Reduce/Reduce Conflict

What Happens
if Choose

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow Expr - Expr$

$Expr \rightarrow (Expr)$

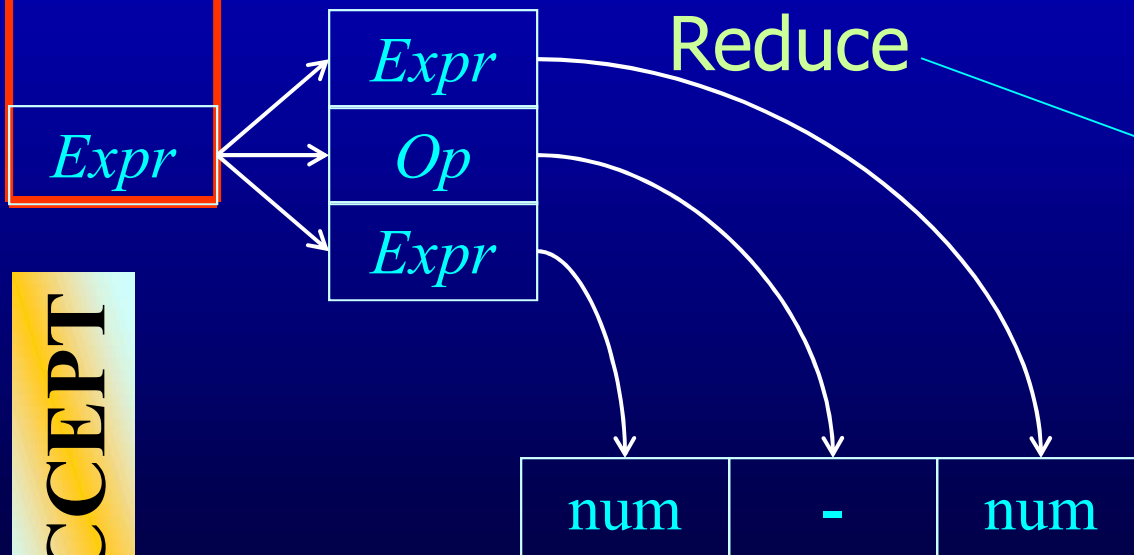
$Expr \rightarrow Expr -$

$Expr \rightarrow num$

$Op \rightarrow +$

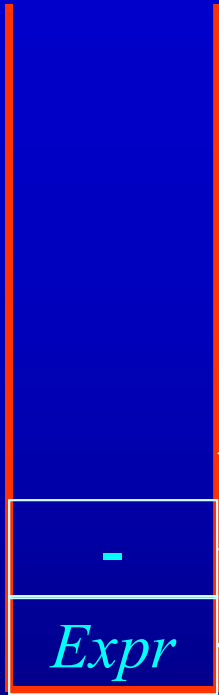
$Op \rightarrow -$

$Op \rightarrow *$



Conflicts

What Happens
if Choose
Shift



SHIFT

num

num

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow Expr - Expr$

$Expr \rightarrow (Expr)$

$Expr \rightarrow Expr -$

$Expr \rightarrow num$

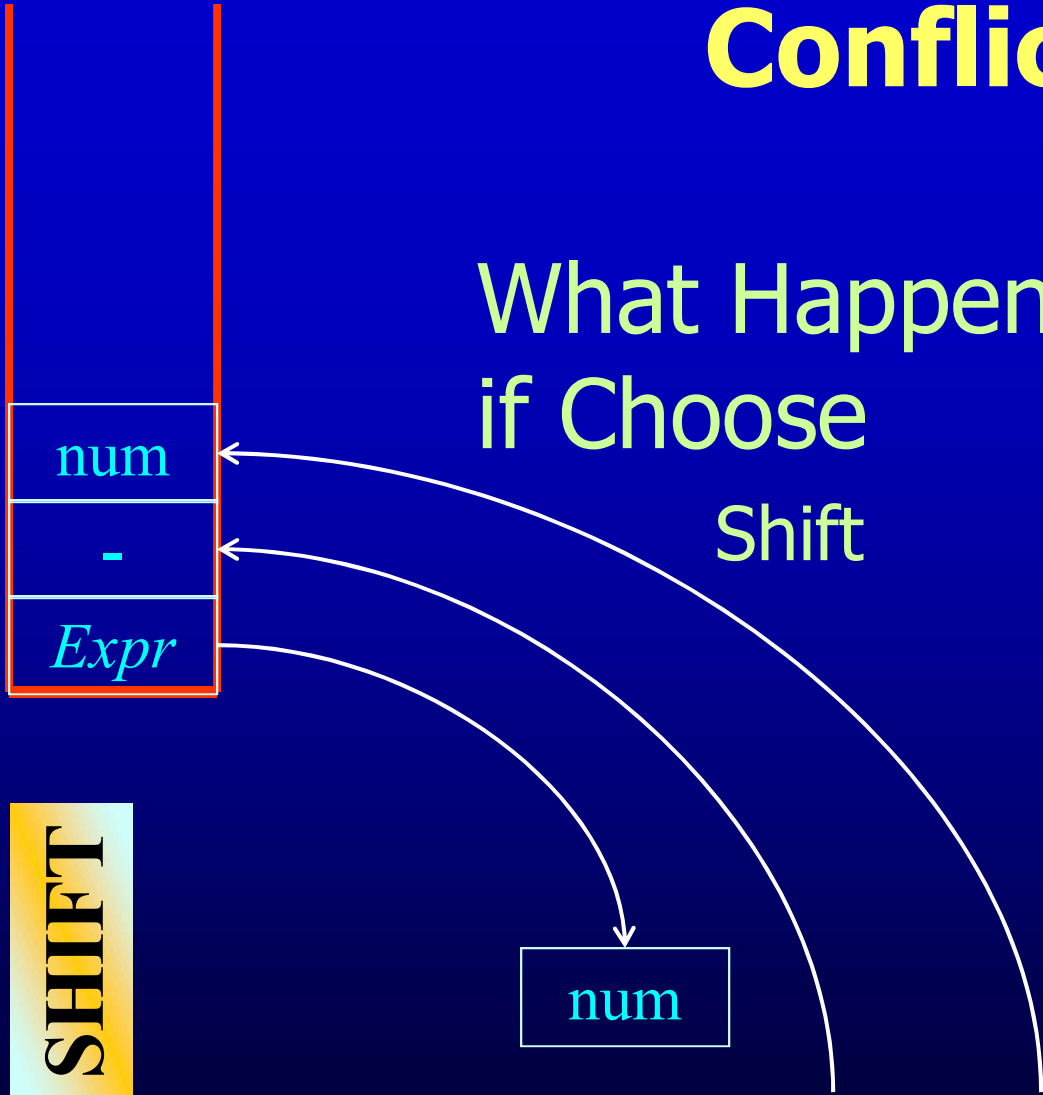
$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$

Conflicts

What Happens
if Choose
Shift



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$Expr \rightarrow Expr - Expr$

$Expr \rightarrow (Expr)$

$Expr \rightarrow Expr -$

$Expr \rightarrow num$

$Op \rightarrow +$

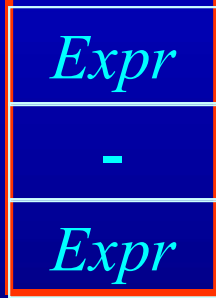
$Op \rightarrow -$

$Op \rightarrow *$

Conflicts

What Happens
if Choose

Shift



REDUCE

num

num

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow Expr - Expr$

$Expr \rightarrow (Expr)$

$Expr \rightarrow Expr -$

$Expr \rightarrow num$

$Op \rightarrow +$

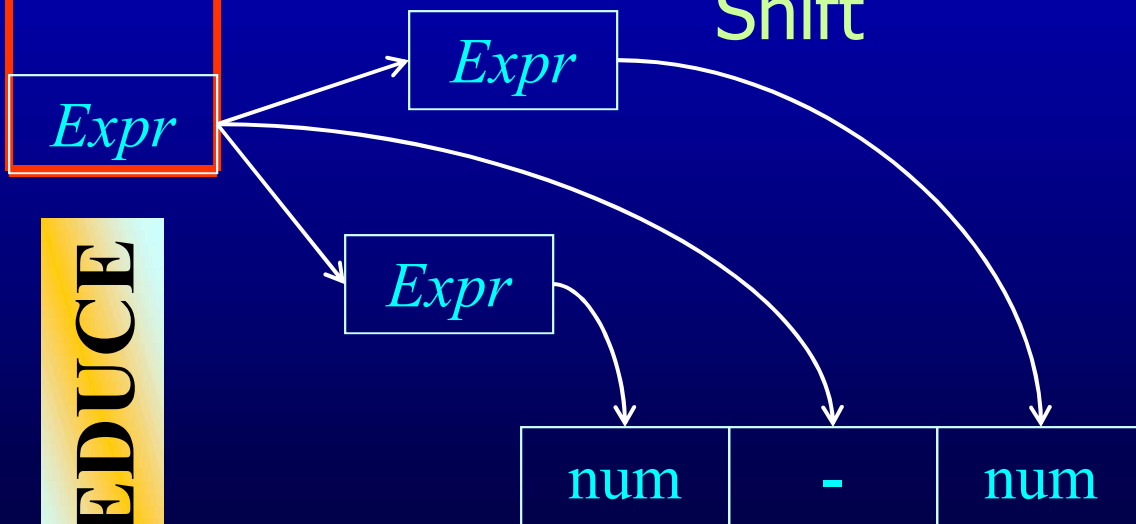
$Op \rightarrow -$

$Op \rightarrow *$

Conflicts

What Happens
if Choose

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$Expr \rightarrow Expr Op Expr$

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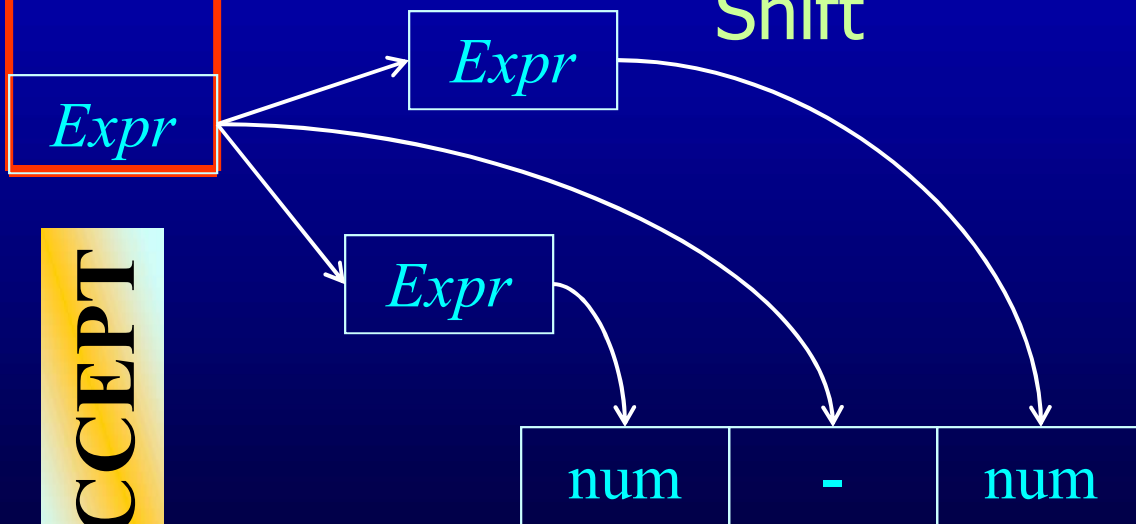
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$Op \rightarrow *$

Conflicts

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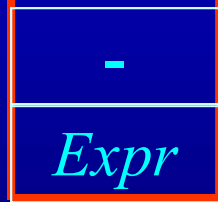
$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$

Shift/Reduce/Reduce Conflict

This Shift/Reduce
Conflict Reflects
Ambiguity in
Grammar



$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow Expr - Expr$

$Expr \rightarrow (Expr)$

$Expr \rightarrow Expr -$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$

Shift/Reduce/Reduce Conflict

This Shift/Reduce Conflict Reflects Ambiguity in Grammar

$$Expr \rightarrow Expr Op Expr$$

~~$$Expr \rightarrow Expr - Expr$$~~

$$Expr \rightarrow (Expr)$$

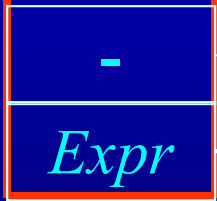
$$Expr \rightarrow Expr -$$

$$Expr \rightarrow num$$

$$Op \rightarrow +$$

$$Op \rightarrow -$$

$$Op \rightarrow *$$



Eliminate by Hacking Grammar



Shift/Reduce/Reduce Conflict

This Shift/Reduce Conflict Can Be Eliminated By Lookahead of One Symbol

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow Expr - Expr$

$Expr \rightarrow (Expr)$

$Expr \rightarrow Expr -$

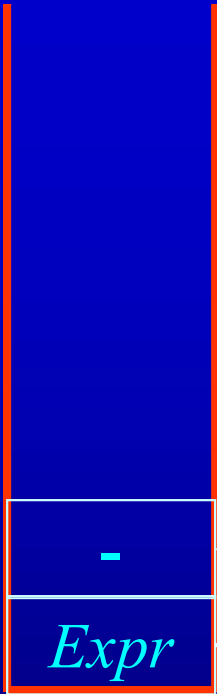
$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$

Parser Generator Should Handle It



Outline

- Ambiguity and Grammar Design
- Bottom-Up parsing
- Conflicts
- **Constructing a Shift-Reduce Parser**
- Parser and syntax trees

Constructing a Parser

- We will construct version with no lookahead
- Key Decisions
 - Shift or Reduce
 - Which Production to Reduce
- Basic Idea
 - Build a DFA to control shift and reduce actions
 - In effect, convert grammar to pushdown automaton
 - Encode finite state control in parse table

Parser State

- Input Token Sequence (\$ for end of input)
- Current State from Finite State Automaton
- Two Stacks
 - State Stack (implements finite state automaton)
 - Symbol Stack (terminals from input and nonterminals from reductions)

Integrating Finite State Control

- Actions
 - Push Symbols and States Onto Stacks
 - Reduce According to a Given Production
 - Accept
- Selected action is a function of
 - Current input symbol
 - Current state of finite state control
- Each action specifies next state
- Implement control using parse table

Parse Tables

State	ACTION			Goto
	()	\$	X
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (3)	reduce (3)	reduce (3)	

- Implements finite state control
- At each step, look up
 - Table[top of state stack] [input symbol]
- Then carry out the action

Parse Table Example

		ACTION		Goto
State	()	\$	X
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (3)	reduce (3)	reduce (3)	

State Stack

Symbol Stack

Input

Grammar

s0

X

(0)

$S \rightarrow X\$$ (1)

$X \rightarrow (X)$ (2)

$X \rightarrow ()$ (3)

Parser Tables

		ACTION		Goto
State	()	\$	X
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (3)	reduce (3)	reduce (3)	

- Shift to s_n
 - Push input token into the symbol stack
 - Push s_n into state stack
 - Advance to next input symbol

Parser Tables

State	ACTION			Goto
	()	\$	X
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (3)	reduce (3)	reduce (3)	

- Reduce (n)
 - Pop both stacks as many times as the number of symbols on the RHS of rule n
 - Push LHS of rule n into symbol stack

Parser Tables

State	ACTION			Goto
	()	\$	X
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (3)	reduce (3)	reduce (3)	

- Reduce (n) (continued)
 - Look up Table[top of the state stack][top of symbol stack]
 - Push that state (in goto part of table) onto state stack

Parser Tables

State	ACTION			Goto
	()	\$	X
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (3)	reduce (3)	reduce (3)	

- Accept
 - Stop parsing and report success

Parse Table In Action

State	ACTION			Goto
	()	\$	X
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (3)	reduce (3)	reduce (3)	

State Stack

Symbol Stack

Input

Grammar

()\$

$S \rightarrow X\$$ (1)

$X \rightarrow (X)$ (2)

$X \rightarrow ()$ (3)

s0

Parse Table In Action

State	ACTION			Goto
	()	\$	X
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (3)	reduce (3)	reduce (3)	

State Stack

Symbol Stack

Input

Grammar

s0

()\$

$S \rightarrow X\$ (1)$

$X \rightarrow (X) (2)$

$X \rightarrow () (3)$

Parse Table In Action

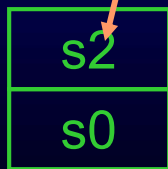
State	ACTION			Goto
	()	\$	
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (3)	reduce (3)	reduce (3)	

State Stack

Symbol Stack

Input

Grammar



(())\$

$S \rightarrow X\$ (1)$

$X \rightarrow (X) (2)$

$X \rightarrow () (3)$

Parse Table In Action

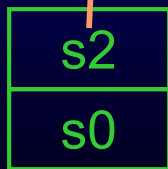
State	ACTION			Goto
	()	\$	X
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (3)	reduce (3)	reduce (3)	

State Stack

Symbol Stack

Input

Grammar



)\$

$S \rightarrow X\$ (1)$

$X \rightarrow (X) (2)$

$X \rightarrow () (3)$

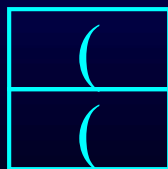
Parse Table In Action

State	ACTION			Goto
	()	\$	
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (3)	reduce (3)	reduce (3)	

State Stack



Symbol Stack



Input

)\$

Grammar

$S \rightarrow X\$$ (1)

$X \rightarrow (X)$ (2)

$X \rightarrow ()$ (3)

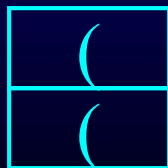
Parse Table In Action

State	ACTION			Goto
	()	\$	
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (3)	reduce (3)	reduce (3)	

State Stack



Symbol Stack



Input

)\$

Grammar

$S \rightarrow X\$$ (1)

$X \rightarrow (X)$ (2)

$X \rightarrow ()$ (3)

Parse Table In Action

State	ACTION			Goto
	()	\$	
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (3)	reduce (3)	reduce (3)	

State Stack



Symbol Stack



Input

)))\$

Grammar

$S \rightarrow X\$$ (1)

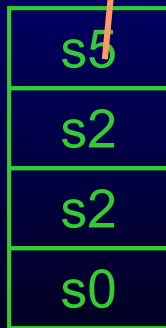
$X \rightarrow (X)$ (2)

$X \rightarrow ()$ (3)

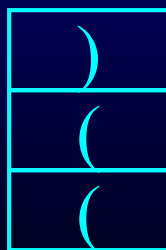
Parse Table In Action

State	ACTION			Goto
	()	\$	
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (3)	reduce (3)	reduce (3)	

State Stack



Symbol Stack



Input

)\$

Grammar

$S \rightarrow X\$$ (1)

$X \rightarrow (X)$ (2)

$X \rightarrow ()$ (3)

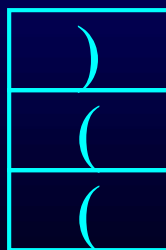
Step One: Pop Stacks

State	ACTION			Goto
	()	\$	
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (3)	reduce (3)	reduce (3)	

State Stack



Symbol Stack



Input

)\$

Grammar

$S \rightarrow X\$$ (1)

$X \rightarrow (X)$ (2)

$X \rightarrow ()$ (3)

Step One: Pop Stacks

State	ACTION			Goto
	()	\$	
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (3)	reduce (3)	reduce (3)	

State Stack

Symbol Stack

Input

Grammar



)\$

$S \rightarrow X\$ (1)$

$X \rightarrow (X) (2)$

$X \rightarrow () (3)$

Step Two: Push Nonterminal

State	ACTION			Goto
	()	\$	
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (3)	reduce (3)	reduce (3)	

State Stack

Symbol Stack

Input

Grammar



)\$

$S \rightarrow X\$ (1)$

$X \rightarrow (X) (2)$

$X \rightarrow () (3)$

Step Two: Push Nonterminal

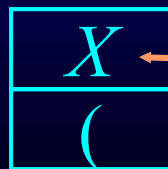
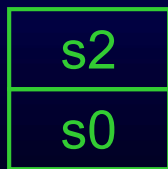
State	ACTION			Goto
	()	\$	X
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (3)	reduce (3)	reduce (3)	

State Stack

Symbol Stack

Input

Grammar



)\$

$S \rightarrow X\$ (1)$

$X \rightarrow (X) (2)$

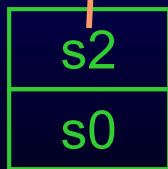
$X \rightarrow () (3)$



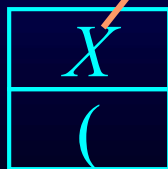
Step Three: Use Goto, Push New State

		ACTION		Goto
State	()	\$	X
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (3)	reduce (3)	reduce (3)	

State Stack



Symbol Stack



Input

)\$

Grammar

$S \rightarrow X\$$ (1)

$X \rightarrow (X)$ (2)

$X \rightarrow ()$ (3)

Step Three: Use Goto, Push New State

State	ACTION			Goto
	()	\$	X
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (3)	reduce (3)	reduce (3)	

State Stack

Symbol Stack

Input

Grammar



)\$

$S \rightarrow X\$ (1)$

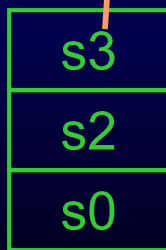
$X \rightarrow (X) (2)$

$X \rightarrow () (3)$

Parse Table In Action

State	ACTION			Goto
	()	\$	X
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (3)	reduce (3)	reduce (3)	

State Stack



Symbol Stack



Input

)\$

Grammar

$S \rightarrow X\$$ (1)

$X \rightarrow (X)$ (2)

$X \rightarrow ()$ (3)

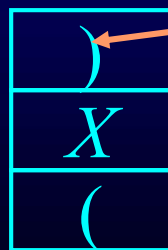
Parse Table In Action

State	ACTION			Goto
	()	\$	X
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (3)	reduce (3)	reduce (3)	

State Stack



Symbol Stack



Input

)\$

Grammar

$S \rightarrow X\$$ (1)

$X \rightarrow (X)$ (2)

$X \rightarrow ()$ (3)

Parse Table In Action

State	ACTION			Goto
	()	\$	X
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (3)	reduce (3)	reduce (3)	

State Stack



Symbol Stack



Input

\$

Grammar

$S \rightarrow X\$ (1)$

$X \rightarrow (X) (2)$

$X \rightarrow () (3)$

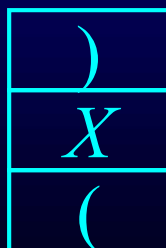
Step One: Pop Stacks

State	ACTION			Goto
	()	\$	X
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (3)	reduce (3)	reduce (3)	

State Stack



Symbol Stack



Input

\$

Grammar

$S \rightarrow X\$ (1)$

$X \rightarrow (X) (2)$

$X \rightarrow () (3)$

Step One: Pop Stacks

State	ACTION			Goto
	()	\$	X
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (3)	reduce (3)	reduce (3)	

State Stack Symbol Stack Input Grammar

\$

$S \rightarrow X\$ (1)$

$X \rightarrow (X) (2)$

$X \rightarrow () (3)$

s0

Step Two: Push Nonterminal

State	ACTION			Goto
	()	\$	X
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (3)	reduce (3)	reduce (3)	

State Stack

Symbol Stack

Input

Grammar

\$

$S \rightarrow X\$ (1)$

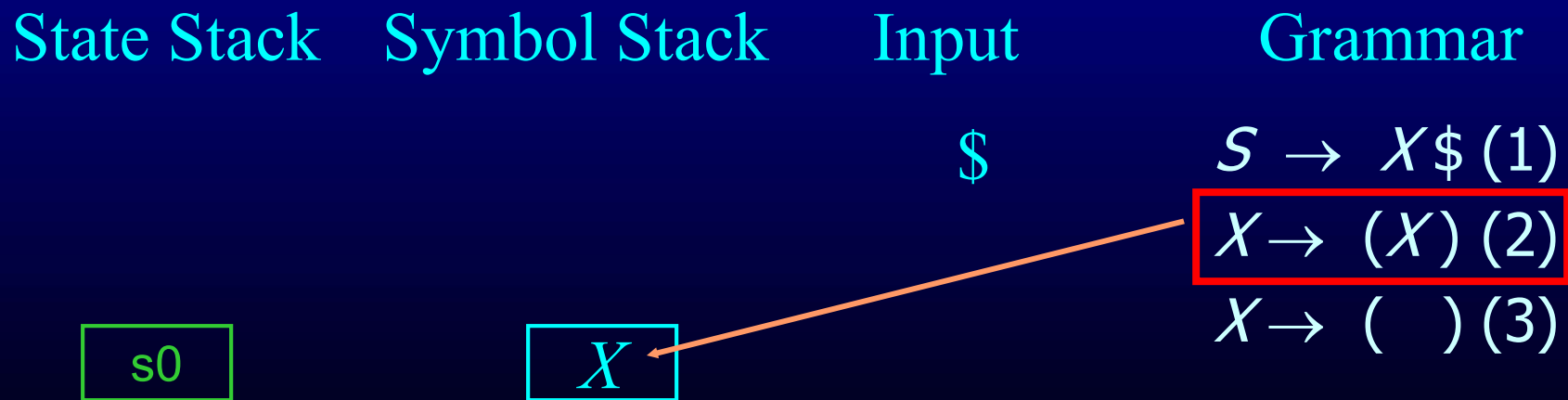
$X \rightarrow (X) (2)$

$X \rightarrow () (3)$

s0

Step Two: Push Nonterminal

State	ACTION			Goto
	()	\$	X
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (3)	reduce (3)	reduce (3)	



Step Three: Use Goto, Push New State

State	ACTION			Goto
	()	\$	X
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (3)	reduce (3)	reduce (3)	

State Stack

Symbol Stack

Input

Grammar

s0

X

\$

$S \rightarrow X\$ (1)$

$X \rightarrow (X) (2)$

$X \rightarrow () (3)$

Step Three: Use Goto, Push New State

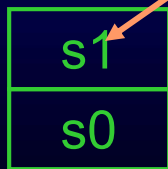
State	ACTION			Goto
	()	\$	X
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (3)	reduce (3)	reduce (3)	

State Stack

Symbol Stack

Input

Grammar



\$

$S \rightarrow X\$ (1)$

$X \rightarrow (X) (2)$

$X \rightarrow () (3)$

Accept the String!

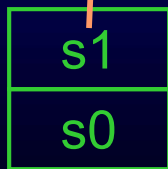
State	ACTION			Goto
	()	\$	
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (3)	reduce (3)	reduce (3)	

State Stack

Symbol Stack

Input

Grammar



\$

$S \rightarrow X\$ (1)$

$X \rightarrow (X) (2)$

$X \rightarrow () (3)$

Key Concepts

- Pushdown automaton for parsing
 - Stack, Finite state control
 - Parse actions: shift, reduce, accept
- Parse table for controlling parser actions
 - Indexed by parser state and input symbol
 - Entries specify action and next state
 - Use state stack to help control
- Parse tree construction
 - Reads input from left to right
 - Bottom-up construction of parse tree

Outline

- Ambiguity and Grammar Design
- Bottom-Up parsing
- Conflicts
- Constructing a Shift-Reduce Parser
- **Parser and syntax trees**

Parser

- Converts program into a parse tree
- Can be written by hand
- Or produced automatically by parser generator
 - Accepts a grammar as input
 - Produces a parser as output
- Practical problem
 - Parse tree for hacked grammar is complicated
 - Would like to start with more intuitive parse tree

Solution

- Abstract versus Concrete Syntax
 - Abstract syntax corresponds to “intuitive” way of thinking of structure of program
 - Omits details like superfluous keywords that are there to make the language unambiguous
 - Abstract syntax may be ambiguous
 - Concrete Syntax corresponds to full grammar used to parse the language
- Parsers are often written to produce abstract syntax trees.